Math M607: Representations of Finite Groups
Fall 2024
MWF 9:10-10:00, BH 242

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Office hours: Monday 10-11, Thursday 1:30-2:30, and by appointment

Texts: Lang, *Algebra*, Revised Third Edition, Chapters 17 (sections 1-5) and 18 (sections 1-8)
Serre, *Linear Representations of Finite Groups*, Parts II and III

Prerequisites: You should know what a module is, and know the statement of the classification of modules over a PID. The language of exact sequences, tensor products, and elementary category theory will be introduced, but briefly, so if you have not had prior exposure to these, you should put in a bit of extra work.

Course Announcement: Representations of groups are an important tool in group theory, number theory, geometry, and topology. A group representation is a group homomorphism $G \to \mathrm{Aut}(M)$ where M is some sort of algebraic structure such as a vector space or an abelian group. Usually M is a module over a ring R, in which case a group representation is the same as an module over the group ring RG. If M is a finitely generated free R-module, then a group representation is the same thing as a group homomorphism $G \to GL_n(R)$. This interplay between noncommutative ring theory, group theory, and linear algebra gives the subject its depth.

The ultimate goal is when G is finite and $R = \mathbb{Z}$, especially the case of a a projective $\mathbb{Z}G$-module. But the journey is long, one first must understand the classical case of $R = \mathbb{C}$, then the more subtle cases of $R = \mathbb{R}$ and $R = \mathbb{Q}$. But to journey all the way to $\mathbb{Z}G$, one must understand the modular case $\mathbb{F}_p$ which is delicate when p divides the order of G. One then lifts this to the p-adic case $R = \mathbb{Z}_p$ and then patches the p-adic and rational cases along the p-adic numbers $R = \mathbb{Q}_p$ to end up with the integral case $R = \mathbb{Z}$. Applications to topology (and perhaps to number theory) will be discussed.

We will start with the noncommutative ring theory, following Chapter XVII of Lang's *Algebra*. We then move to classical representation theory, following Chapter XVIII of Lang and transitioning to Serre's book *Linear Representations of Finite Groups* which covers the the complex, real, rational, and modular case.

I will lead you on the final part of the journey.