1. ✓Are path components equal to connected components for a CW-complex?

2. Show that a homology equivalence between simply-connected CW-complexes is a homotopy equivalence. Give counterexamples omitting (a) the hypothesis of a CW-complex or (b) the hypothesis of simply-connected.

3. ✓What are examples of based spaces (A, a_0) and (B, b_0) so that

$$[(A, a_0), (B, b_0)] \to [A, B]$$

is not a bijection. Give some criteria for this to be a bijection. Do you know what happens for the case when A is a sphere?

4. ✓Show that a map $X \to Y$ gives a map on CW approximations $X' \to Y'$ so that

$$\begin{array}{ccc} X' & \longrightarrow & Y' \\ \downarrow & & \downarrow \\ X & \longrightarrow & Y \end{array}$$

commutes up to homotopy. Show that that top horizontal map is well-defined up to homotopy.

5. ✓Let the G be the dihedral group of order 6 (= symmetric group S_3). Then G acts on the solid equilateral triangle by isometries. Give the triangle a G-CW-structure (or at least, describe the cells).

6. ✓Give a map $f : A \to B$ which is not a cofibration by showing the definition is not satisfied.

7. ✓Show that a pair (X, A) is a cofibration iff $A \times I \cup X \times 0 \subset X \times I$ is a retract.

8. ✓Use obstruction theory to show (at least in the simply-connected case), that a CW pair (X, A) is a cofibration.

9. ✓Let

$$\begin{array}{ccc} A & \longrightarrow & B \\ \downarrow & & \downarrow \\ C & \longrightarrow & D \end{array}$$

be a pushout diagram which remains a pushout diagram after crossing with I. Show that if $A \to C$ is a cofibration then so is $B \to D$.

10. ✓Let X be a G-set. Suppose H is a subgroup of G. Give, with proof, a bijection $\mathrm{Map}_G(G/H, X) \leftrightarrow X^H$.

11. Let $f : (B, b_0) \to (Y, y_0)$ be a map of based spaces. Establish a long exact sequence of pointed sets[1]

$$\cdots \to \pi_n(B, b_0) \to \pi_n(Y, y_0) \to \pi_n(f, b_0, y_0) \to \pi_{n-1}(B, b_0) \to \ldots$$

12. ✓$c \in \mathcal{C}$ is a *representing object* for a functor $F : \mathcal{C}^{\mathrm{op}} \to \mathrm{Set}$ if there is a natural isomorphism $F(-) \cong \mathcal{C}(-, c)$. Show that any two representing objects are isomorphic. Give an example of a functor with more than one representing object.

13. ✓Show that the fundamental group of an aspherical manifold is torsionfree.

14. ✓Are the categories Set and $\mathrm{Set}^{\mathrm{op}}$ isomorphic?

15. ✓"Adjoints are unique:" Show that if $F \dashv G$ and $F \dashv G'$, then $G \cong G'$.

16. "Adjoints are everywhere:" Find left adjoints for the forgetful functors G-Set $\to$ Set and ComRing $\to$ Set, and find an adjoint pair of your own.

17. ✓Express the property $\mathrm{Hom}(A \otimes_R B, C) \cong \mathrm{Hom}_R(A, \mathrm{Hom}(B, C))$ as an adjoint pair for a noncommutative ring R, or express the property $\mathrm{Map}(A \times_G B, C) \cong \mathrm{Map}_G(A, \mathrm{Map}(B, C))$ as an adjoint pair for a group G.

18. ✓How many objects in $\mathrm{Or}(S_3)$? How many morphisms in $\mathrm{Or}(S_3)$?

19. ✓Show that $\mathrm{Aut}_{\mathrm{Or}\, G}(G/H, G/H) \cong N(H)/H$ where $N(H) = \{g \in G \mid gHg^{-1} = H\}$ is the normalizer of H.

[1] A sequence of pointed sets $(X, x_0) \xrightarrow{\alpha} (Y, y_0) \xrightarrow{\beta} (Z, z_0)$ is exact if $\beta^{-1} z_0 = \alpha(X)$.

20. Show $\mathbb{Z}[1/2] = \operatorname*{colim}_{n\to\infty} \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \to \dots$ by showing

$$\begin{array}{ccccccc} \mathbb{Z} & \xrightarrow{\times 2} & \mathbb{Z} & \xrightarrow{\times 2} & \mathbb{Z} & \longrightarrow & \cdots \\ & {\scriptstyle\phi_0}\searrow & \downarrow{\scriptstyle\phi_1} & \swarrow{\scriptstyle\phi_2} & & & \\ & & \mathbb{Z}[\frac{1}{2}] & & & & \end{array}$$

is a universal cocone (i.e. satisfies the universal property of a colimit), where $\phi_n(a) = a/2^n$.

21. ✓*Stability:* Let

$$\begin{array}{ccccccc} X_0 & \xrightarrow{f_0} & X_1 & \xrightarrow{f_1} & X_1 & \longrightarrow & \cdots \\ & {\scriptstyle\phi_0}\searrow & \downarrow{\scriptstyle\phi_1} & \swarrow{\scriptstyle\phi_2} & & & \\ & & \operatorname*{colim}_{n\to\infty} X_n & & & & \end{array}$$

be a classical colimit. Show

(a) $\operatorname*{colim}_{n\to\infty} X_n = \bigcup_n \phi_n(X_n)$

(b) Let $a, b \in X_n$. Then $\phi_n(a) = \phi_n(b) \iff \exists N,$ such that im $a =$ im $b \in X_N$.

22. Let $X(0)_\bullet \to X(1)_\bullet \to X(2)_\bullet \to \cdots$ be a sequence of chain complexes and chain maps. Use *stability* to prove *homology commutes with classical colimits*:

$$\operatorname*{colim}_{n\to\infty} H_i(X(n)_\bullet) \xrightarrow{\cong} H_i(\operatorname*{colim}_{n\to\infty} X(n)_\bullet)$$

23. ✓The p-adic integers $\mathbb{Z}_p = \varprojlim \mathbb{Z}/p^n$ are uncountable.

24. ✓Let $F : G \to G$ be a group automorphism. Then F is inner if and only if $\mathrm{Id} \simeq F$, i.e. the functors Id and F are naturally isomorphic.

25. Define a simplicial set with X_n the monotone maps from $[n] = \{0, 1, 2, \dots, n\}$ to $[1] = \{0, 1\}$. For each n, how many degenerate and nondegenerate simplicies are there in X_n? Show that the geometric realization of $|X_.|$ is the interval.

26. ✓Show $|\mathcal{N}_\bullet C_2)| \cong \mathbb{R}P^\infty$.

27. Let $\omega = 0 \to 1 \to 2 \to \cdots$. Compute $H_n(\omega; M) = H_n(P_{\underline{\mathbb{Z}}} \otimes M)$.

28. ✓Give an alternate characterization of a free ω-CW-complex. (Only the answer is required.).

29. ✓Let $F, G : \mathcal{C} \to \mathcal{D}$ be functors. There is a natural transformation $F \Rightarrow G$ if and only if there is a functor $H : \mathcal{C} \times (0 < 1) \to \mathcal{D}$ with $F = H_{\mathcal{C}\times 0}$ and $G = H_{\mathcal{C}\times 1}$.

30. Let Λ be the category $2 \leftarrow 0 \to 1$. Given an alternate characterization of a free Λ-CW-complex.

31. (a) Let $c_u : R \to R$ be an inner automorphism. Then $K_0(c_u)$ is the identity.

    (b) There is a functor $K_0 : \mathrm{Or}\, G \to \mathrm{Ab}$ so that $K_0(G/H) = K_0(\mathbb{Z}H)$ and if $K < H < G$, then $K_0(G/K \to G/H) = K_0(\mathbb{Z}K \hookrightarrow \mathbb{Z}H)$.

32. If $F : \mathcal{A} \to \mathcal{B}$ is an equivalence of small additive categories, then $K_0 F : K_0\mathcal{A} \to K_0\mathcal{B}$ is an isomorphism.