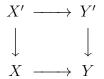
- 1. \checkmark Are path components equal to connected components for a CW-complex?
- Show that a homology equivalence between simply-connected CW-complexes is a homotopy equivalence. Give counterexamples omitting

 (a) the hypothesis of a CW-complex or
 (b) the hypothesis of simply-connected.
- 3. \checkmark What are examples of based spaces (A, a_0) and (B, b_0) so that

$$[(A, a_0), (B, b_0)] \rightarrow [A, B]$$

is not a bijection. Give some criteria for this to be a bijection. Do you know what happens for the case when A is a sphere?

4. \checkmark Show that a map $X \to Y$ gives a map on CW approximations $X' \to Y'$ so that



commutes up to homotopy. Show that that top horizontal map is welldefined up to homotopy.

- 5. \checkmark Let the G be the dihedral group of order 6 (= symmetric group S_3). Then G acts on the solid equilateral triangle by isometries. Give the triangle a G-CW-structure (or at least, describe the cells).
- 6. \checkmark Give a map $f : A \rightarrow B$ which is not a cofibration by showing the definition is not satisfied.
- 7. \checkmark Show that a pair (X, A) is a cofibration iff $A \times I \cup X \times 0 \subset X \times I$ is a retract.
- 8. \checkmark Use obstruction theory to show (at least in the simply-connected case), that a CW pair (X, A) is a cofibration.
- 9. $\checkmark \text{Let}$



be a pushout diagram which remains a pushout diagram after crossing with I. Show that if $A \to C$ is a cofibration then so is $B \to D$.

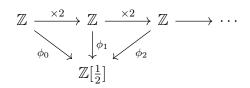
- 10. \checkmark Let X be a G-set. Suppose H is a subgroup of G. Give, with proof, a bijection Map_G(G/H, X) $\leftrightarrow X^{H}$.
- 11. Let $f: (B, b_0) \to (Y, y_0)$ be a map of based spaces. Establish a long exact sequence of pointed sets¹

 $\cdots \to \pi_n(B, b_0) \to \pi_n(Y, y_0) \to \pi_n(f, b_0, y_0) \to \pi_{n-1}(B, b_0) \to \ldots$

- 12. $\checkmark c \in \mathcal{C}$ is a representing object for a functor $F : \mathcal{C}^{\text{op}} \to \text{Set}$ if there is a natural isomorphism $F(-) \cong \mathcal{C}(-, c)$. Show that any two representing objects are isomorphic. Give an example of a functor with more than one representing object.
- 13. \checkmark Show that the fundamental group of an aspherical manifold is torsion free.
- 14. \checkmark Are the categories Set and Set^{op} isomorphic?
- 15. \checkmark "Adjoints are unique:" Show that if $F \dashv G$ and $F \dashv G'$, then $G \cong G'$.
- 16. "Adjoints are everywhere:" Find left adjoints for the forgetful functors G-Set \rightarrow Set and ComRing \rightarrow Set, and find an adjoint pair of your own.
- 17. \checkmark Express the property $\operatorname{Hom}(A \otimes_R B, C) \cong \operatorname{Hom}_R(A, \operatorname{Hom}(B, C))$ as an adjoint pair for a noncommutative ring R, or express the property $\operatorname{Map}(A \times_G B, C) \cong \operatorname{Map}_G(A, \operatorname{Map}(B, C))$ as an adjoint pair for a group G.
- 18. \checkmark How many objects in $Or(S_3)$? How many morphisms in $Or(S_3)$?
- 19. \checkmark Show that $\operatorname{Aut}_{\operatorname{Or} G}(G/H, G/H) \cong N(H)/H$ where $N(H) = \{g \in G \mid gHg^{-1} = H\}$ is the normalizer of H.

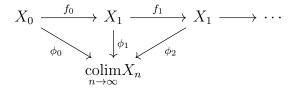
¹A sequence of pointed sets $(X, x_0) \xrightarrow{\alpha} (Y, y_0) \xrightarrow{\beta} (Z, z_0)$ is exact if $\beta^{-1} z_0 = \alpha(X)$.

20. Show $\mathbb{Z}[1/2] = \operatorname{colim}_{n \to \infty} \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \to \dots$ by showing



is a universal cocone (i.e. satisfies the universal property of a colimit), where $\phi_n(a) = a/2^n$.

21. \checkmark Stability: Let



be a classical colimit. Show

- (a) $\underset{n \to \infty}{\operatorname{colim}} X_n = \bigcup_n \phi_n(X_n)$
- (b) Let $a, b \in X_n$. Then $\phi_n(a) = \phi_n(b) \iff \exists N$, such that im $a = \text{im } b \in X_N$.
- 22. Let $X(0)_{\bullet} \to X(1)_{\bullet} \to X(2)_{\bullet} \to \cdots$ be a sequence of chain complexes and chain maps. Use *stability* to prove *homology commutes with classical colimits*:

$$\operatorname{colim}_{n \to \infty} H_i(X(n)_{\bullet}) \xrightarrow{\cong} H_i(\operatorname{colim}_{n \to \infty} X(n)_{\bullet})$$

- 23. \checkmark The *p*-adic integers $\mathbb{Z}_p = \lim_{\longleftarrow} \mathbb{Z}/p^n$ are uncountable.
- 24. \checkmark Let $F : G \to G$ be a group automorphism. Then F is inner if and only if Id $\simeq F$, i.e. the functors Id and F are naturally isomorphic.
- 25. Define a simplicial set with X_n the monotone maps from $[n] = \{0, 1, 2, ..., n\}$ to $[1] = \{0, 1\}$. For each n, how many degenerate and nondegenerate simplicies are there in X_n ? Show that the geometric realization of $|X_{\cdot}|$ is the interval.

- 26. \checkmark Show $|\mathcal{N}_{\bullet}C_2)| \cong \mathbb{R}P^{\infty}$.
- 27. Let $\omega = 0 \to 1 \to 2 \to \cdots$. Compute $H_n(\omega; M) = H_n(P_{\mathbb{Z}} \otimes M)$.
- 28. \checkmark Give an alternate characterization of a free ω -CW-complex. (Only the answer is required.).
- 29. \checkmark Let $F, G : \mathcal{C} \to \mathcal{D}$ be functors. There is a natural transformation $F \Rightarrow G$ if and only if there is a functor $H : \mathcal{C} \times (0 < 1) \to \mathcal{D}$ with $F = H_{\mathcal{C} \times 0}$ and $G = H_{\mathcal{C} \times 1}$.
- 30. Let Λ be the category $2 \leftarrow 0 \rightarrow 1$. Given an alternate characterization of a free Λ -CW-complex.
- 31. (a) Let $c_u : R \to R$ be an inner automorphism. Then $K_0(c_u)$ is the identity.
 - (b) There is a functor K_0 : Or $G \to Ab$ so that $K_0(G/H) = K_0(\mathbb{Z}H)$ and if K < H < G, then $K_0(G/K \to G/H) = K_0(\mathbb{Z}K \hookrightarrow \mathbb{Z}H)$.
- 32. If $F : \mathcal{A} \to \mathcal{B}$ is an equivalence of small additive categories, then $K_0F : K_0\mathcal{A} \to K_0\mathcal{B}$ is an isomorphism.