Projects for M623

This document is a work in progress! Eventually all students will chose a topic and lecture on it. You will make up, administer, and grade a quiz on your topic.

1. Immersion theory. State the Hirsch-Smale immersion theorem. Show that every n-manifold immerses in $\mathbb{R}^{2n}$. Turn a $2-$sphere inside out. Classify immersions of S^n in $\mathbb{R}^{2n}$. Show that a parallelizable n-manifold immerses in $\mathbb{R}^{2n+1}$. (Ref: Look at the statement (not the proof) of Theorem 7.35 of Algebraic and Geometric Surgery and then apply obstruction theory.) [Chao]

2. Exotic spheres I. Milnor's original article and the Eells-Kuiper invariant. [Juanita]

3. Exotic spheres II. Plumbing. (See Browder's book and manifold atlas) Explain how, in every dimension $4k$ with $k > 1$, to construct a compact parallelizable $4k$-manifold with signature 8 and boundary a homotopy sphere. Explain why the boundary is an exotic sphere. Include a discussion of the poincare homology 3-sphere. [Ash]

4. Exotic spheres III. Brieskorn varieties (See Hirzebruch's survey)

5. topological 4-manifolds - concentrating on the simply-connected case. (See Wild World of 4-manifolds and talk to Margaret Doig) [Henry]

6. Applications of the h-cobordism theorem. State the h-cobordism theorem and s-cobordism theorem. Give (some) of the applications in Milnor's book. Show concordance of smooth structures implies isotopy and diffeomorphism. Show that if $(W; M_0, M_1)$ is an h-cobordism that $W - M_1 \cong M_0 \times [0, 1)$ and that $M_0 \times \mathbb{R} \cong M_1 \times \mathbb{R}$. Perhaps discuss

Milnor's paper "Two complexes which are homeomorphic but combinatorially distinct" or Mazur's result that if M and N are homotopy and stable tangential equivalent then $M \times \mathbb{R}^3 \cong M \times \mathbb{R}^3$. [Tristen]

Show that if M and N are homotopy equivalent closed manifolds with stable trivial tangent bundle then $M \times S^n$ is diffeomorphic to $N \times S^n$ for sufficiently large n(?) Prove concordance implies isotopy (?) and concordance implies diffeomorphism. [Tristan]

7. Smoothing theory (see Davis-Petrosyan) [Tarek?]

8. Spherical Space Form problem [Ryan]

9. Poincare complexes [Justin?]

10. Borel and Novikov conjectures. [Hailiang]

    (a) Borel Conjecture: State and prove the homotopy rigidity statement that two aspherical CW complexes with isomorphic fundamental group are homotopy equivalent. Then the speaker should state a geometric rigidity statement such as Mostow rigidity. The speaker should carefully state the Borel conjecture.

    (b) Discuss L-classes for PL-manifolds. Show PL-invariance.

    (c) State the Novikov Conjecture. Give an interpretation in the case of $\mathbb{Z}^n$.

    (d) It would be good if the Borel and Novikov Conjectures could be translated into the framework of surgery theory.

11. Quadratic forms over rings with involution. Give both Wall's and Ranicki's definition of quadratic form. Define the even-dimensional L-groups and the even-dimensional surgery obstruction. [Shida]

12. Arf/Kervaire invariant. Define the Arf invariant. Discuss connections with homotopy spheres and homotopy theory. Discuss π_2^S and the Lie invariant framing on the torus. [Deniz]

13. Failure of Whitney trick in dimension 4 (see Kervaire-Milnor) [Neal?]