

Outline of Math 623; Topics in Geometric Topology: Manifold Theory

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MWF 10:10-11:00 in RH104

The basic topic will be the classification of manifolds with a focus on surgery theory; the classification of high dimensional manifolds. I will, however, cover topics of interest to low-dimensional topologists and algebraic topologists.

The prerequisites are Math 521 and 522, in other words singular homology theory, CW complexes, fundamental group, and covering spaces. But familiarity with the cohomology ring, homotopy groups, and the definition of a differentiable manifold and its tangent bundle would help, as would some of Chapter 4 of Davis-Kirk.

Here are topics.

1. Types of manifolds: smooth, PL, topological. (Reference: Davis-Petrosyan)
2. Handlebody theory, Morse theory, and surgery. (References: Milnor, A procedure . . . and Kosinski, Differentiable Manifolds)
3. Bundle theory (References: Davis-Kirk, Davis-Petrosyan)
4. Exotic Spheres I. There are three constructions: sphere bundles (see Milnor, On manifolds . . .), plumbing (see Browder, also Ranicki), and Brieskorn spheres (see Hirzebruch)
5. Embeddings and Immersions: The Whitney embedding theorem, the Whitney trick, the Hirsch-Smale theorem, turning the 2-sphere inside out. (see Ranicki).

6. Algebraic K -theory (Reference: Milnor, Whitehead torsion)
7. Degree one normal maps, Poincaré duality, and the surgery exact sequence (Reference: Lück, A basic . . .)
8. Exotic Spheres II (Reference: Kervaire-Milnor)
9. Spherical Space Forms (Reference: Davis-Milgram)
10. Aspherical manifolds (Reference: Lück, A survey on aspherical manifolds)

The Bible of surgery theory is Wall's book: [Surgery on compact manifolds](#). But unfortunately, this book is difficult even for experts. So instead, focus on the references below:

References

- [1] Browder, [Surgery on simply-connected manifolds](#)
- [2] Davis and Kirk, [Lecture notes in algebraic topology](#)
- [3] Davis and Milgram, [A survey of the spherical space form problem](#)
- [4] Davis and Petrosyan, [Manifolds and Poincaré complexes](#)
- [5] Hirzebruch, [Singularities and exotic spheres](#)
- [6] Kervaire and Milnor, [Groups of homotopy spheres. I](#)
- [7] Kosinski, [Differentiable Manifolds](#)
- [8] Lück, [A basic introduction to surgery theory](#)
- [9] Lück, [A survey on aspherical manifolds](#)
- [10] Milnor, [A procedure for killing the homotopy groups of differentiable manifolds](#)
- [11] Milnor, [On manifolds homeomorphic to the 7-sphere](#)
- [12] Milnor, [Whitehead torsion](#)
- [13] Ranicki, [Algebraic and geometric surgery](#)