The proof of Corollary 8.3 in my paper [1] is a bit garbled. Here is a clarification of the proof.

For a closed odd-dimensional oriented manifold M define the *rational semicharacteristic*

$$\hat{\chi}_{1/2}(M;\mathbb{Q}) = \sum_{i<\dim M/2} (-1)^i \dim H_i(M;\mathbb{Q}) \in \mathbb{Z}/2$$

Lemma 8.2. If $\mathbb{Z}_2 \times \mathbb{Z}_2$ acts freely, preserving orientation on a closed manifold M^{4k+1}, then the rational semicharacteristic is even.

Corollary 8.3. If a 2-group π acts freely, preserving orientation on a closed manifold M^{4k+1} with odd rational semicharacteristic, then π is cyclic.

Proof. The proof is by induction on π. Suppose π is a 2-group whose order is at least 8. Let H be an index two subgroup. By induction H is cyclic. Let $h \in H$ have order 2. Then for any $\mathbb{Q}H$-module V, the dimension of the -1 eigenspace of $h : V \to V$ is even, since $\mathbb{R} \otimes_{\mathbb{Q}} V$ admits a complex structure. Note $H_i(M;\mathbb{Q}) = H_i(M;\mathbb{Q})_+ \oplus H_i(M;\mathbb{Q})_-$ where $\pm$ refers to the ± 1-eigenspaces of the $\langle h \rangle$-action. Furthermore, a transfer argument shows that $H_i(M;\mathbb{Q})_+ \cong H_i(M/\langle h \rangle;\mathbb{Q})$.

Thus if M^{4k+1} is a free G-manifold with odd rational semicharacteristic, then $M^{4k+1}/\langle h \rangle$ is a free $\pi/\langle h \rangle$ manifold with odd rational semicharacteristic. Hence $\pi/\langle h \rangle$ is cyclic. It follows that π is cyclic. □

References

[1] James F. Davis, "The Surgery Semicharacteristic," Proceedings of the London Mathematical Society, (3) 47 (1983), 411–428.