

One can ask a more general version of question 26.2. Recall that for a discrete group G , one denotes by \underline{EG} any G -CW complex which has the property that for any subgroup H of G

$$(\underline{EG})^H = \begin{cases} \emptyset & |H| = \infty \\ \simeq * & |H| < \infty \end{cases}$$

Then for any G -CW complex X all of whose isotropy groups are finite, there is a G -map $X \rightarrow \underline{EG}$, which is unique up to G -homotopy. In particular, any two models for \underline{EG} are G -homotopy equivalent. For the mapping class group Γ_g , it is well-known that Teichmüller space τ_g is a model for $\underline{E}\Gamma_g$, although I don't know the best reference (the key result that fixed point sets of finite subgroups are nonempty is due to Kerckhoff [2]). It also seems clear to the experts that there is a cocompact model (see [1]).

Here is a modified version of Question 26.2.

QUESTION Is there a finite-dimensional model of $\underline{E}\Gamma_g$ which has a contractible compactification which is small at infinity and equivariant with respect to the mapping class group? Is there a compact model with the same properties?

If there is such a cocompact model, then the assembly maps with respect to the family of finite subgroups in algebraic K - and L -theory would be injective by Rosenthal's thesis. If there is only such a finite-dimensional model, then it is likely the same conclusion holds, but Rosenthal's thesis would have to be modified.

References

- [1] L. Ji and S. A. Wolpert. A cofinite universal space for proper actions for mapping class groups. arXiv:0811.3871.
- [2] S. P. Kerckhoff. The Nielsen realization problem. *Ann. of Math. (2)* 117 (1983), 235–265.