26

ISOMORPHISM CONJECTURES FOR THE MAPPING CLASS GROUP

by James F. DAVIS

I enjoyed Guido's lecture in Münster where he showed that Teichmüller space is a classifying space for actions of the mapping class group whose isotropy groups are all finite. I will pose a question and then recall one I posed at that time.

Let Σ_g be a closed surface of genus g, let Γ_g be its mapping class group, and $\tau_g \cong \mathbf{R}^{6g-6}$ its Teichmüller space.

QUESTION 26.1 (A special case of the Borel Conjecture). *Let Γ be a torsion free subgroup of the mapping class group, for example the Torelli group. Is the action of Γ on τ_g topologically rigid ? That is, is any proper homotopy equivalence of manifolds $h\colon M \to \tau_g/\Gamma$, which is a homeomorphism outside of a compact set, homotopic to a homeomorphism, via a homotopy which is constant outside of a compact set ?*

QUESTION 26.2 (Isomorphism conjecture injectivity). *Is there a contractible compactification of Teichmüller space which is small at infinity and equivariant with respect to the action of the mapping class group ?*

To say a compactification $\overline{\tau}_g$ of τ_g is *small at infinity* means that for every compact subset K of Teichmüller space, for every boundary point $y \in \overline{\tau}_g - \tau_g$, and for every neighborhood U of y in the compactification $\overline{\tau}_g$, there is a smaller neighborhood V so that for every $\gamma \in \Gamma_g$, if $\gamma K \cap V$ is nonempty, then $\gamma K \subset U$.

DISCUSSION. A solution to Question 26.1 would likely involve carrying out the program of Farrell–Jones [1] in the mapping class group case. A positive solution to Question 26.2 would likely lead to a proof of the injectivity map of the assembly map in K- and L-theory with respect to the family of finite subgroups (by modifying Rosenthal's thesis [3]), and thereby a new proof of the Novikov conjecture in this case. (It seems that the Novikov conjecture in the mapping class group case has been recently proved by Ursula Hamenstädt [2].)

REFERENCES

[1] FARRELL, F. T. and L. E. JONES. Isomorphism conjectures in algebraic K-theory. *J. Amer. Math. Soc. 6* (1993), 249–297.

[2] HAMENSTÄDT, U. Geometry of the mapping class groups I : Boundary amenability. Preprint arXiv : math.GR/0510116 (2005–2008).

[3] ROSENTHAL, D. Splitting with continuous control in algebraic K-theory. *K-Theory 32* (2004), 139–166.

James F. Davis

Department of Mathematics
Indiana University
Bloomington, Indiana 47405
USA
e-mail : jfdavis@indiana.edu

Achevé de composer le 13 mai 2008 à 19 : 04